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# Status of Relativity with observer-independent length and velocity scales[1]

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**ABSTRACT**

I have recently shown that it is possible to formulate the Relativity postulates in a way that does not lead to inconsistencies in the case of space-times whose structure is governed by observer-independent scales of both velocity and length. Here I give an update on the status of this proposal, including a brief review of some very recent developments. I also emphasize the role that one of the $\kappa$-Poincaré Hopf algebras could play in the realization of a particular example of the new type of postulates. I show that the new ideas on Relativity require us to extend the set of tools provided by $\kappa$-Poincaré and to revise our understanding of certain already available tools, such as the energy-momentum coproduct.

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# 1 Relativity and observer-independent scales

In these notes I examine the status of my recent proposal [1, 2] attempting to identify consistent Relativity postulates that involve both an observer-independent velocity scale ($c \sim 3{\cdot}10^8 m/s$) and an observer-independent length scale ($L_p \sim 1.6{\cdot}10^{-35} m$). Readers already familiar with the proposal [1, 2] might find anyway useful my review for what concerns the results obtained in Refs. [3, 4, 5, 6] which were motivated by Refs. [1, 2] and provided important contributions to the programme.

I start with a few remarks on the motivation for exploring the possibility that the Relativity postulates might involve an observer-independent length scale, in addition to the now familiar observer-independent velocity scale $c$. The fact that the Planck length $L_p$ is proportional to both $\hbar$, the Planck constant, and $G$, the gravitational constant ($L_p \equiv \sqrt{\hbar G/c^3}$), appears to invite one to speculate that $L_p$ might play a role in the microscopic (possibly quantum) structure of space-time, and in fact many "quantum-gravity" theories [7, 8] have either assumed or stumbled upon this possibility. However, a fundamental role for $L_p$ in the structure of space-time appears to be conceptually troublesome for one of the cornerstones of Einstein's Special Relativity: FitzGerald-Lorentz length contraction. The Relativity Principle demands that physical laws should be the same in all inertial frames, including the law that would attribute to the Planck length a fundamental role in the structure of space-time, whereas, according to FitzGerald-Lorentz length contraction, different inertial observers would attribute different values to the same physical length. If the Planck length only has the role we presently attribute to it, which is basically the role of a coupling constant (an appropriately rescaled version of the coupling $G$), no problem arises for FitzGerald-Lorentz contraction, but if we try to promote $L_p$ to the status of an intrinsic characteristic of space-time structure (or a characteristic of the kinematic rules that govern particle propagation in space-time) it is natural to find conflicts with FitzGerald-Lorentz contraction.

For example, it is very hard (perhaps even impossible) to construct discretized versions or non-commutative versions of Minkowski space-time which enjoy ordinary Lorentz symmetry.[2] Discretization length scales and/or non-commutativity length scales naturally end up acquiring different values for different inertial observers, just as one would expect in light of the mechanism of FitzGerald-Lorentz contraction. Therefore, unless the Relativity postulates are modified, it appears impossible to attribute to the Planck length a truly fundamental (observer-independent) intrinsic role in the microscopic structure of space-time.

We are of course not forced to introduce such a modification of the Relativity postulates. In fact, we do not (yet?) have any data that require us to attribute to $L_p$ an observer-independent role in the microscopic structure of space-time (note, however, the intruiging indications emerging from the data analysed in Ref. [14] and references therein) and the theoretical arguments suggesting such a role are still rather debatable (see, however, the related comments reported here in Section 4). On the other hand, just because such an hypothesis is fully consistent with presently available data (the presently accepted version of the Relativity postulates has been succesfully tested

[2] Pedagogical illustrative examples of this observation have been discussed, *e.g.*, in Ref. [9] for the case of discretization and in Refs. [10, 11, 12, 13] for the case of non-commutativity.

only up to scales that are insufficient for probing the Planckian regime) and because some, however preliminary, supporting theoretical arguments have been found, it is of course legitimate to explore the possibility that indeed the fundamentally correct formulation of the Relativity postulates might involve another observer-independent scale, in addition to $c$.

In order to prepare the ground for the line of analysis advocated in these notes (and previously advocated in Refs. [1, 2]) it si convenient to review the role that observer-independent scales (or absence thereof) already played in Galilean Relativity and Einstein's Special Relativity. The Relativity Principle demands that "the laws of physics are the same in all inertial frames" and clearly the implications of this principle for space-time structure and kinematics depend very strongly on whether there are fundamental scales of velocity and/or length. In fact, the introduction of a fundamental scale is itself a physical law, and therefore the Relativity Principle allows the introduction of such fundamental scales only if the rules that relate the observations performed by different inertial observers are structured in such a way that all inertial observers can agree on the value and physical interpretation of the fundamental scales. The Galileo/Newton rules of transformation between inertial observers can be easily obtained by combining the Relativity Principle with the assumption that there are no observer-independent scales for velocity or length. For example, without an observer-independent velocity scale, there is no plausible alternative [1] to the simple Galilean law $v' = v_0 + v$ of composition of velocities.

Special Relativity describes the implications of the Relativity Principle for the case in which there is an observer-independent velocity scale. Einstein's second postulate can be naturally divided in two parts: the introduction of an observer-independent velocity scale $c$ and the proposal of a physical interpretation of $c$ as the speed of light. This second postulate, when combined with the Relativity Principle (which is the first postulate of Special Relativity) and with the additional assumption that there is no observer-independent length scale leads straightforwardly to the now familiar Lorentz transformations, with their associated familiar formulation of FitzGerald-Lorentz contraction. The assumption that there is no observer-independent length scale plays a key role already in the way in which the second postulate was stated. Experimental data available when Special Relativity was formulated, such as the ones of the Michelson-Morley experiments, only concerned light of very long wavelengths (extremely long in comparison with the length scale $L_p$ introduced by Planck a few years earlier) and therefore the second postulate could have accordingly attributed to $c$ the physical role of speed of long-wavelength light (the infinite-wavelength limit of the speed of light); however, the implicit assumption of absence of an observer-independent length scale allowed to extrapolate from Michelson-Morley data a property for light of all wavelengths. In fact, it is not possible to assign a wavelength dependence to the speed of light without introducing either a "preferred" class of inertial frames or an observer-independent length scale.

All the revolutionary elements of Special Relativity (in comparison with the Relativity of Galileo and Newton) are easily understood as direct consequences of the introduction of an observer-independent velocity scale. This is particularly clear for the deformed law of composition of velocities, $v' = (v_0 + v)/(1 + v_0 v/c^2)$, and the demise of absolute time (an absolute concept of time is untenable when an observer-independent velocity scale governs the exchange of information between clocks).

Within the perspective here being adopted it is clear that the Planck-length problem I am concerned with can be described as the task of showing that the Relativity Principle can coexist with some types of postulates stating that the fundamental structure

of space-time involves both an observer-independent velocity scale $c$ and an observer-independent length scale $L_p$. The addition of an observer-independent length scale does not require major revisions of the physical interpretation of $c$, but, because of the mentioned connection between wavelength independence and absence of an observer-independent length scale, I shall not authomatically assume that it is legitimate to extrapolate from our long-wavelength data:

- (law1): The value of the fundamental velocity scale $c$ can be measured by each inertial observer as the $\lambda/L_p \to \infty$ limit of the speed of light of wavelength $\lambda$.

While for $c$ we can at least rely on long-wavelength data, we basically have no experimental information on the role (if any) of $L_p$ in space-time structure. The type of exploratory research programme I have proposed [1, 2] must therefore naturally start by identifying some examples of postulates that are logically consistent and involve both $c$ and $L_p$ as observer-independent scales. Eventually one would like this programme to evolve to the point where all such logically consistent formulations of Relativity are identified, and then leave to experimentalists the final task of establishing which (if any) of these candidates is realized in Nature. Since this research programme is just starting off, I felt [1, 2] that it would be appropriate to focus on one specific illustrative example of new postulates, analysing it in depth so that we could be reassured that the set of such logically consistent formulations of Relativity is non-empty. This illustrative example of new Relativity postulates is introduced in the next Section, and is also the main focus of most of the remainder of these notes.

# 2 An illustrative example of the new type of Relativity postulates

As anticipated in the preceding Section, in this Section I will focus on one example of new Relativity postulates. For the framework I am advocating the only ingredient on which we lack experimental guidance is the role to be attributed in the postulates to $L_p$. The other parts of the postulates have in fact already been discussed in the preceding Section: the new theory will maintain the Relativity Principle, it will introduce both $c$ and $L_p$ as observer independent scales in the postulates, and it will attribute to $c$ the physical role of the long-wavelength limit of the speed of light.

In choosing an illustrative example of postulate attributing a role to $L_p$ in space-time structure and kinematics, I found [1, 2] appropriate to give priority to ideas that would have significant phenomenological consequences (so that I could show explicitly that the issue I am considering is not merely of academic interest) and that can make some contact with preliminary indications of quantum-gravity theories. As I shall here discuss in greater detail in Sections 4 and 5 the hypothesis that the conventional dispersion relation $E^2 = c^2p^2$ be deformed at the Planck scale finds some motivation in recent quantum-gravity theoretical studies and can lead to new effects that are small enough to be consistent with all presently-available data, while being large enough to be tested in the near future. In the following I will also argue that, within the new type of Relativity theory which I am proposing, a deformation of the dispersion relation can also be connected with the emergence of a minimum length, another popular "quantum-gravity idea".

Motivated by these considerations, in Refs. [1, 2] I chose to use the following illustrative example of postulate attributing a role in space-time structure and kinematics to a length scale $\tilde{L}_p$:

- (law2): Each inertial observer can establish the value of $\tilde{L}_p$ (same value for all inertial observers) by determining the dispersion relation for photons, which takes the form $E^2 = c^2p^2 + f(E, p; \tilde{L}_p)$, where the function $f$ has leading $\tilde{L}_p$ dependence given by: $f(E, p; L_p) \simeq \tilde{L}_p c E p^2$.

Of course, at the conceptual level we should even contemplate the possibility that $\tilde{L}_p$ be completely unrelated to $L_p$ (we cannot exclude the existence of a completely new length scale in the correct formulation of the Relativity postulates), but in light of the considerations reported above it appears reasonable to explore in particular the possibility the quantity[3] setting the strength of the dispersion-relation deformation and the Planck length calculated *a la* Planck be identified up to a numerical coefficient not too different from 1 and a possible sign choice ($\tilde{L}_p \equiv \rho L_p$, with $\rho \in R$, $|\rho| \sim 1$).

I must now check the logical consistency of a Relativity theory based on (law1) and (law2), and attempt to extract its most characteristic features.

### 2.1 Transformation rules (one-particle case)

The logical consistency of the new postulates (law1) and (law2) requires that, in their analyses of photon data in leading order in $\tilde{L}_p$, all inertial observers agree on the dispersion relation $E^2 = c^2p^2 + \tilde{L}_p c p^2 E$, for fixed (observer-independent) values of $c$ and $\tilde{L}_p$. The postulates do not explicitly concern massive particles, which are at rest ($p = 0$) in some inertial frames and in those frames have a "rest energy" which we indentify with the mass $c^2m$. For massive particles I tentatively adopt the dispersion relation $E^2 = c^4m^2 + c^2p^2 + \tilde{L}_p c p^2 E$, which satisfies these properties. I postpone to future studies the possibility that these postulates might coexist with more complicated dispersion relations for massive particles of the type $E^2 = c^4m^2 + c^2p^2 + \tilde{L}_p c p^2 E + F(p, E; m; \tilde{L}_p)$, which are consistent with (law2) not only in the case $F = 0$ (here considered) but also whenever $F$ is such that $F(p, E; 0; \tilde{L}_p) = F(p, E; m; 0) = F(0, E; m; \tilde{L}_p) = 0$.

Let me therefore assume $E^2 = c^4m^2 + c^2p^2 + \tilde{L}_p c p^2 E$ and look for boost generators (generators of rotations clearly do not require modification) for which this dispersion relation is an invariant (indeed valid for all inertial observers). At this stage (see the wording adopted in (law2)) we shall be satisfied with checking logical consistency at leading order in $\tilde{L}_p$. For additional simplicity, here let me also limit[4] my considerations

[3] As illustrated by the specific example (law2), the observer-independent length scale must not necessarily have the physical meaning of the length of something. For example, as indeed it happens in (law2), the role of $L_p$ in space-time structure could be such that it provides a sort of reference scale for momenta (wavelengths).

[4] More general boosts can then be constructed by insisting [1] on ordinary rotational invariance of the theory, which still holds.

to boosts along the $z$ direction of particles with momentum only in the $z$ direction, so that I am only to enforce invariance of $E^2 = c^4m^2 + c^2p_z^2 + \tilde{L}_p c p_z^2 E$. The Lorentz $z$-boost generator, $B_z = icp_z \partial/\partial E + ic^{-1}E\partial/\partial p_z$, clearly requires a deformation. I make the ansatz $B_z^{\tilde{L}_p} = i[cp_z + \tilde{L}_p\Delta_E]\partial/\partial E + i[E/c + \tilde{L}_p\Delta_{p_z}]\partial/\partial p_z$, for which one easily finds that the sought invariance translates into the requirement $2E\Delta_E - 2p_z\Delta_{p_z} = -2E^2p_z - p_z^3$. The simplest solutions are of the type $2\Delta_E = 0$ , $\Delta_{p_z} = E^2 + p_z^2/2$ and $\Delta_{p_z} = 0$ , $\Delta_E = -Ep_z - p_z^3/(2E)$. Various arguments of simplicity [1] (including considerations involving combinations of boosts and rotations and the desire to have generators which would be well-behaved even off shell) lead me to adopt the first option, so the new $z$-boost generator takes the form

$$B_z^{\tilde{L}_p} = icp_z \frac{\partial}{\partial E} + i[E/c - \tilde{L}_p E^2/c^2 - \tilde{L}_p p_z^2/2]\frac{\partial}{\partial p_z} \ . \tag{1}$$

One important observation to be made at this point is that the generators of boosts (and rotations) constructed in the way I just described turn out to correspond to the leading-order-in-$\tilde{L}_p$ version of the Lorentz-sector generators of a well-known $\kappa$-Poincaré Hopf algebra [10, 11, 13], the example of $\kappa$-Poincaré Hopf algebra first introduced in Ref. [12]. In this sense just like the introduction of the Special Relativity postulates led to preexisting Lorentz-group mathematics, the example of new Relativity postulates I am analyzing leads to preexisting $\kappa$-Poincaré mathematics (note however that some of the observations reported in Subsection 2.3 do not fit in the $\kappa$-Poincaré mathematics, at least not in the way in which it is presently understood).

For brevity, here I do not note the formulas for finite transformations. Having obtained the new generators of boosts and rotations one immediately obtains infinitesimal transformations (*e.g.*, $dE/d\xi = i[B_z^{\tilde{L}_p}, E] = -cp_z$ , $dp_z/d\xi = i[B_z^{\tilde{L}_p}, p_z] = -E/c + \tilde{L}_p E^2/c^2 + \tilde{L}_p p_z^2/2$), and then finite transformations are obtained by straightforward (but tedious) integration. The interested reader can find this discussion, including explicit formulas for finite transformations, in Ref. [1].

## 2.2 Length contraction

The example of new postulates I am focusing on makes a non-trivial assumption about energy-momentum space: even in the Planck regime energy-momentum space is classical (although deformed). This is a plausible, but strong, assumption, which of course is reasonable to consider, especially in light of the exploratory attitude of these first studies of new Relativity postulates. It might however be too much to assume that also the space-time sector remains classical. In this respect it is particularly important that in the preceding Subsection I was led to generators which had already emerged in preexisting $\kappa$-Poincaré mathematics; in fact, the relevant Hopf algebra has been understood [12, 13] as being dual to a non-commutative space-time, the $\kappa$-Minkowski space-time ($l, m = 1, 2, 3$):

$$[x_m, t] = i\tilde{L}_p x_m \ , \quad [x_m, x_l] = 0 \ . \tag{2}$$

This fact that the space-time counter-part of the energy-momentum space which appears in the postulates might be "quantum" invites one to be prudent [1] in making

considerations on the space-time picture of the transformation rules imposed by the illustrative example of new Relativity postulates on which I am focusing. We can however obtain some (partial) information on the nature of this space-time sector even just using structures obtained in energy-momentum space. This is the task that I reserved for the present Subsection. My observations concern the general topic of "relativistic length contraction", considering both wavelengths (momenta) and lengths.

One first observation concerns the possible emergence of a minimum wavelength (maximum momentum). Let us consider a photon which, for a given inertial observer, is moving along the positive direction of the $z$ axis with momentum $p_0$ (and, of course, as imposed by the new dispersion relation, has energy $E_0 \simeq p_0 + \tilde{L}_p c p_0^2/2$). The new relativity postulates imply [1] that for another inertial observer, which the first observer sees moving along the same $z$ axis, the photon has momentum $p$ related to $p_0$ by

$$p = p_0 e^{-\xi} + \tilde{L}_p p_0^2 e^{-\xi} - \tilde{L}_p p_0^2 e^{-2\xi} \ . \tag{3}$$

Ordinary Lorentz boosts are of course obtained as the $\tilde{L}_p \to 0$ limit of the new boosts (3). The comparison between (3) and its $\tilde{L}_p \to 0$ limit provides some insight on the type of deformation of FitzGerald-Lorentz contraction that characterizes the new postulates. As long as $p_0 < 1/|\tilde{L}_p|$ (wavelength $\lambda_0 > |\tilde{L}_p|$) and $e^{-\xi} \ll 1/(|\tilde{L}_p| p_0)$ the relation between $p$ and $p_0$ is well described by ordinary Lorentz transformations. Within this analysis in leading order in $\tilde{L}_p$ it is not legitimate to consider the case $e^{-\xi} > 1/(|\tilde{L}_p| p_0)$ (which would require an exact all-order analysis of the implications of the function $f(E,p;\tilde{L}_p)$ introduced in the postulates), but we can look at the behaviour of the transformation rules when $e^{-\xi}$ is smaller but not much smaller than $1/(|\tilde{L}_p| p_0)$. While the transformation rules are basically unmodified when $e^{-\xi} \ll 1/(|\tilde{L}_p| p_0)$, as $e^{-\xi}$ approaches from below the value $1/(|\tilde{L}_p| p_0)$ the transformation rules are more and more severely modified: for large boosts, the ones that would lead to nearly Planckian wavelengths in the ordinary special-relativistic case, the magnitude of the wavelength contraction is significantly modified. The modification takes the form of a reduction of the contraction if $\tilde{L}_p > 0$. For example, taking indeed $\tilde{L}_p > 0$, for $e^{-\xi} \simeq 1/(3\tilde{L}_p p_0)$ one would ordinarily predict $p \simeq 1/(3\tilde{L}_p)$ while the new transformation rules predict the softer momentum $p \simeq 2/(9\tilde{L}_p)$. This suggests that there should exist an exact all-order form of $f(E,p;\tilde{L}_p)$ (extending the present $f(E,p;\tilde{L}_p) \simeq \eta L_p c E p^2$ leading-order analysis) such that when one inertial observer assigns to the photon momentum smaller than $1/\tilde{L}_p$ (wavelength greater than $\tilde{L}_p$) all other inertial observers also find momentum smaller than $1/\tilde{L}_p$. It would then be possible to consider as unphysical momenta greater than $1/\tilde{L}_p$. $\tilde{L}_p$ would have the role of observer-independent minimum wavelength. This would not be very surprising for a Relativity with observer-independent $\tilde{L}_p$ and $c$. $\tilde{L}_p$ would be the observer-independent minimum wavelength, just in the same sense that $c$ is the observer-independent maximum speed in ordinary Special Relativity (speeds greater than $c$ are unphysical and a velocity which is smaller than $c$ for one inertial observer is also smaller than $c$ for all other inertial observers).

Similar findings emerge from the analysis of length contraction within the illustrative example of new Relativity theory on which I am focusing. This can be shown by

analysing a gedanken length-measurement procedure. A key point for this observation is the fact that the dispersion relation $E^2 \simeq c^2 p^2 + \tilde{L}_p c E p^2$ corresponds[5] to the deformed speed-of-light law

$$v_\gamma = \frac{dE}{dp} = c\,(1 + \tilde{L}_p c^{-1} E) \ . \tag{4}$$

The wavelength dependence of the speed-of-light law (4) can lead to the emergence of a minimum length in measurement analysis. In order to see this, let us consider two observers each with its own (space-) ship moving in the same space direction, the $z$-axis, with different velocities (*i.e.* with some relative velocity), and let us mark "A" and "B" two $z$-axis points on one of the ships (the rest frame). The procedure of measurement of the distance $AB$ is structured as a time-of-flight measurement: an ideal mirror is placed at B and the distance is measured as the half of the time needed by a first photon wave packet, centered at momentum $p_0$, sent from A toward B to be back at A (after reflection by the mirror). Timing is provided by a digital light-clock: another mirror is placed in a point "C" of the rest frame/ship, with the same $z$-axis coordinate of A at some distance $AC$, and a second identical wave packet, again centered at $p_0$, is bounced back and forth between A and C. The rest-frame observer will therefore measure $AB$ as $AB' = v_\gamma(p_0) \cdot N \cdot \tau_0 / 2$, where $N$ is the number of ticks done by the digital light-clock during the A→B→A journey of the first wave packet and $\tau_0$ is the time interval corresponding to each tick of the light-clock ($\tau_0 = 2\,AC/v_\gamma(p_0)$). The observer on the second (space-) ship, moving with velocity $V$ with respect to the rest frame, will instead attribute to $AB$ the value

$$AB'' = \frac{v_\gamma(p)^2 - V^2}{v_\gamma(p)} N \frac{\tau}{2} \ , \tag{5}$$

where $p$ is related to $p_0$ through (3) and $\tau$ is the time interval which the second observer, moving with respect to the rest frame, attributes to each tick of the light-clock. It is easy to verify that $\tau$ is related to $\tau_0$ by

$$\tau = \frac{v_\gamma(p_0)}{\sqrt{v_\gamma(p')^2 - V^2}} \tau_0 \ , \tag{6}$$

where $p'$ is related to $p_0$ through the formula for boosts in a direction orthogonal to the one of motion of the photon. Combining (5) and (6) one easily obtains

$$AB'' = \frac{[v_\gamma(p)^2 - V^2] v_\gamma(p_0)}{v_\gamma(p)\sqrt{v_\gamma(p')^2 - V^2}} N \frac{\tau_0}{2} = \frac{v_\gamma(p)^2 - V^2}{v_\gamma(p)\sqrt{v_\gamma(p')^2 - V^2}} AB' \ . \tag{7}$$

The implications of (7) for length contraction are in general quite complicated, but they are easily analyzed in both the small-$V$ and the large-$V$ limits (examined here of course

[5] The careful reader will realize that by assuming that the relation $v = dE/dp$ is unmodified I am actually stating a (perhaps not very strong) property of the space-time sector. From the results obtained in Ref. [15] one can conclude that this property is enjoyed by the space-time of (2).

in leading order in $L_p$). For small $V$ and small momentum (large wavelength) of the probes Eq. (7) reproduces ordinary FitzGerald-Lorentz contraction. For large $V$ Eq. (7) predicts that $AB''$ receives two most important contributions: the familiar FitzGerald-Lorentz term ($AB' \cdot \sqrt{c^2 - V^2}$) and a new term which is of order $\tilde{L}_p |p| AB'/\sqrt{c^2 - V^2}$. As $V$ increases the ordinary FitzGerald-Lorentz contribution to $AB''$ decreases as usual, but the magnitude of the new correction term increases. Imposing[6] $|p| > |\delta p| > 1/AB''$ (the probe wavelength must of course be shorter than the distance being measured) one arrives at the result $AB'' > \sqrt{c^2 - V^2} AB' + \tilde{L}_p AB'/(AB'' \sqrt{c^2 - V^2})$. For $\tilde{L}_p$ positive this result clearly implies that $AB'' > \tilde{L}_p$ for all values of $V$. Again I must remind the reader that I am working in leading order in $\tilde{L}_p$, and therefore the results cannot be trusted when $V$ is large enough that the correction term is actually bigger than the 0-th order contribution to $AB''$, but we can trust the indications of this analysis as long as the correction is smaller than the 0-th order term, and in that regime one finds that (for positive $\tilde{L}_p$) FitsGerald-Lorentz contraction is being significantly softened in the region corresponding to nearly Planckian contraction. This result clearly supports the hypothesis that there should exist a consistent all-order form of $f(E, p; \tilde{L}_p)$ such that when one inertial observer assigns to a length value greater than $\tilde{L}_p$ all other inertial observers also find that length to be greater than $\tilde{L}_p$. Such a form of $f(E, p; \tilde{L}_p)$ would provide a relativistic theory with observer-independent scales $c$ and $\tilde{L}_p$ in which $\tilde{L}_p$ has the intuitive role of "minimum length" described above.

### 2.3 Kinematical conditions for particle-production processes

As I emphasized in Ref. [1], an important requirement for the logical consistency of a Relativity theory is that the laws imposed on particle-production processes should be the same in all inertial frames, *i.e.* all observers should agree on whether or not a certain particle-production process is allowed. This requirement is trivially satisfied in ordinary Special Relativity. Let me discuss this in the simple case of a scattering process $a+b \rightarrow c+d$ (collision processes with incoming particles $a$ and $b$ and outgoing particles $c$ and $d$). Also in this Subsection for simplicity I focus on the case of one space dimension (the generalization to multi-dimensional spaces is described in Ref. [1]) and work to leading order in $\tilde{L}_p$. The special-relativistic kinematic requirements for such processes are $E_a + E_b - E_c - E_d = 0$ and $p_a + p_b - p_c - p_d = 0$, and, using the special-relativistic transformation rules, $dE_j/d\xi = -p_j$, $dp_j/d\xi = -E_j$, one immediately verifies that when the requirements are satisfied in one inertial frame they are also verified in all other inertial frames [1].

The fact that the energy-momentum transformation rules imposed by the postulates (law1),(law2) are non-linear (unlike special-relativistic transformation rules) provides

[6] The careful reader will realize that actually one does not even need to impose $|p| > 1/AB''$ in order to find evidence of saturation of length contraction. The new term of the form $|\tilde{L}_p||p|AB'/\sqrt{c^2 - V^2}$ (for $\tilde{L}_p > 0$) increases with increasing $V$ for two reasons: because the denominator $\sqrt{c^2 - V^2}$ gets smaller and because the numerator gets larger (larger $V$ means larger boost and therefore larger $p$). One easily then finds that at some point (for some value of $V$) the correction term is actually larger than the 0th-order term $\sqrt{c^2 - V^2} AB'$. This is the point where my leading-order analysis stops to be reliable, but clearly the result suggests that length-contraction is saturating to a minimum length.

room for various alternatives for the laws to be satisfied by particle-production processes. This point is discussed in detail in Ref. [1]. Here I just want to mention two possibilities whose consistency with the postulates has been already verified. The first example is

- (cons): $a + b \to c + d$ collision processes must satisfy the requirements

$$E_a + E_b - \tilde{L}_p c p_a p_b - E_c - E_d + \tilde{L}_p c p_c p_d = 0 \ , \tag{8}$$

$$p_a + p_b - \tilde{L}_p (E_a p_b + E_b p_a)/c - p_c - p_d + \tilde{L}_p (E_c p_d + E_d p_c)/c = 0 \ . \tag{9}$$

It is easy to verify, using (1), that these conditions are satisfied in all inertial frames if they are satisfied in one of them. The particle-production law (cons) is a rather natural manifestation of the observer-independent scale $\tilde{L}_p$. The difference between the special-relativistic particle-production laws and the laws (cons) reflects the introduction of $\tilde{L}_p$ in the postulates, just like the difference between the Galilean velocity-composition law, $v' = v_0 + v$, and the special-relativistic velocity-composition law, $v' = (v_0 + v)/(1 + v_0 v/c^2)$, reflects the introduction of $c$ in the postulates.

The type of non-linearity encoded in the postulates (law1),(law2) is also consistent with a completely different alternative type of particle-production laws. Whereas (cons) is a "single-channel conservation law", just like its counter-part in Special Relativity, it is also possible to find consistent multi-channel laws for particle production. A significant example is

- (cons'): $a + b \to c + d$ collision processes must satisfy one of the requirements obtained by permutations of $p_a, p_b, \tilde{p}_c, \tilde{p}_d$ in the conditions

$$E_a + E_b - E_c - E_d = 0 \ , \tag{10}$$

$$p_a \dot{+} p_b \dot{+} \tilde{p}_c \dot{+} \tilde{p}_d = 0 \ , \tag{11}$$

where the deformed sum $\dot{+}$ is defined by $k \dot{+} q \equiv k + q + \tilde{L}_p E_k q$ and $\tilde{k} \equiv -k - \tilde{L}_p E_k k$ (with $E_k$ denoting the energy that corresponds to the momentum $k$).

It is again easy to verify, using (1), that the particle-production law (cons') is satisfied in all inertial frames if it is satisfied in one of them. It is somewhat more difficult to build some intuition for this alternative possibility which can be introduced consistently with the illustrative example of new Relativity postulates on which I am focusing. However, if the expectation that the space-time sector is described by (2) is correct, the law (cons') is actually to be favoured. In fact, in Ref. [5] it was shown that the construction of a consistent theory on the space-time (2) leads to the law (cons').

The content of (cons') is not as shocking as it may seem at first sight. It says that, in the case of a $a + b \to c + d$ process, there are 24 channels available to the process (associated with the 24 possible permutations of the particle momenta $p_a, p_b, \tilde{p}_c, \tilde{p}_d$). The process will be allowed whenever one of the 24 cases is satisfied. What can be somewhat shocking is that this type of structure does not allow to describe the particle-production laws as laws of conservation of energy-momentum (since the particles can choose between 24 different conditions, none of these conditions can acquire the special status of a, deformed, law of energy-momentum conservation). Reassuringly in the limit in which the particles have energies (and momenta) much smaller than $1/|\tilde{L}_p|$ (the only limit in which we presently have conclusive experimental information on energy-momentum conservation) all 24 channels collapse into a single energy-momentum conservation condition, the one of Special Relativity.

## 2.4 Postulates beyond leading order

The results described up to this point are the ones on which I based[7] my proposal [1, 2] of Relativity postulates with more than one observer-independent scale. The analysis was done in leading order in $\tilde{L}_p$, leaving for future studies the search of consistent choices of the all-order function $f(E, p; \tilde{L}_p)$ that appears in the postulate (law2). As mentioned in Subsection 2.2 (and in Refs. [1, 2]) an important hint appears to come from the fact that my analyses led to leading-order expressions for the generators of boosts and rotations that are recognizable as the leading-order approximation of the generators in the Lorentz sector of the example of $\kappa$-Poincaré Hopf algebra proposed in Ref. [12]. The connection between the new Relativity postulates and this Hopf algebra was further explored, within an all-order analysis, by Kowalski-Glikman [3] who adopted as a natural candidate for the function $f(E, p; \tilde{L}_p)$ the one which is inferred from the all-order form of the relevant $\kappa$-Poincaré "$M^2$" casimir (*i.e.* in analogy with my leading-order proposal). Specifically, using the form of this casimir, Kowalski-Glikman provided an all-order generalization[8] of one of the arguments (here reviewed in Subsection 2.2) which I used [2] in support of the emergence of a minimum wavelength (maximum momentum) in the illustrative example of new Relativity postulates considered here and in Refs. [1, 2]. Bruno, Kowalski-Glikman and I also showed, in a very recent study [6], that the use of the relevant $\kappa$-Poincaré "$M^2$" casimir in the postulates leads to consistent transformation rules between different inertial observers, again generalizing to all orders the leading-order results I reported in Refs. [1, 2]. An all-order formulation of the particle-production rules (cons') were reported by Arzano and myself in the very recent Ref. [5].

All these results appear to support my conjecture that the results reported in leading order in Refs. [1, 2] could be straightforwardly generalized to the level of an all-order analysis.

[7] However, the specific formulation (cons') of the second type of particle-production rules is a more recent result obtained in Ref. [5].

[8] Note that in generalizing to all orders the minimum-length proposal I had put forward in Ref.[2], Kowalski-Glikman also observed [3] that upon describing my function $f(E, p; \tilde{L}_p)$ with the $\kappa$-Poincaré "$M^2$" casimir one is led (upon making again the sign choice required for the emergence of a minimum wavelength) to the conclusion that in the infinite-energy limit the speed of massless particles is actually infinite. In my leading-order analysis one could only reliably establish that in the range where the leading order is meaningful the speed of massless particles grows with energy. Kowalski-Glikman's observation that this speed actually diverges asymptotically is fully consistent with the conceptual starting points of my work: if Planckian lengths should not be subject to FitzGerald-Lorentz contraction there must be a particular limit of the theory in which the transformation rules are effectively Galilean (no contraction of Planckian lengths). It is however also important to stress (this was not stressed in Ref. [3]) that the infinite-speed limit is only to be understood as an asymptotic behaviour: any real photon (finite energy) will have a finite speed and transformations between observers using such photons as their probes would not be Galilean. There is no role for real Galilean transformations in the new theory, but the Galilean asymptote is crucial for the overall logical consistency.

# 3 Relation with quantum symmetries

It is natural to assume that the coexistence of the Relativity Principle with length and velocity observer-independent scales should lead to the emergence of space-time symmetries, just in the same sense that Special Relativity, with its single observer-independent scale, leads to Lorentz symmetry. The fact that the new symmetries should involve an additional scale and should reproduce ordinary Lorentz invariance in a certain limit of the additional scale (the $L_p \to 0$ limit) suggests that the subject of "quantum groups" and "quantum algebras" should be in some way relevant. It is probably too early to conclude that this connection should characterize all examples of the type of relativistic theories I proposed (theories in which the Relativity Principle coexists with observer-independent scales of both velocity and length), but it definitely characterizes the specific illustrative example I analyzed in detail here and in Refs. [1, 2]. In fact, as already observed in Section 2, the postulate (law2) involves a dispersion relation which corresponds to the leading-order-in-$\tilde{L}_p$ version of a casimir that has emerged [12, 13] in the quantum-algebra literature, and, upon imposing consistency with the Relativity Principle, I was led to boost (and rotation) generators which can also be recognized as the leading-order-in-$\tilde{L}_p$ version of the generators of the relevant quantum algebra. Like the Special-Relativity postulates provided a possible role in physics for the pre-existing mathematics of the Lorentz group, the illustrative example of Relativity with two observer-independent scales I considered led me to a possible role in physics for the $\kappa$-Poincaré quantum algebra proposed in Refs. [12, 13].

I was unable to find in the mathematics literature the finite new-boost trasformations I obtained in Ref. [1], but, based on comparison with the analysis in Ref. [11] (which concerned a rather similar quantum algebra), I am confident that my results have been derived consistently with the spirit of quantum algebras.

While there is a wide-spread belief (see, *e.g.*, Ref. [11]) that $\kappa$-Poincaré quantum algebras do not have an associated group action (this action should only lead to a "quasi-group" in the sense of Batalin [16]), I have shown in Ref. [1] that the Lorentz sector of the specific $\kappa$-Poincaré algebra [12, 13] that appears to be relevant for my illustrative example of new Relativity postulates does reassuringly lead to ordinary group structure. This follows straightforwardly from the observation that the Lorentz sector of the relevant $\kappa$-Poincaré algebra does satisfy the criteria derived by Batalin [16] for the associated finite transformations to form group.

While the one-particle sector appears to be fully consistent with the mathematics of quantum algebras, the analysis of particle-production processes reported in Section 2 appears to require some new algebraic tools. In particular, at least according to the standard interpretation of the strictly mathematical language of analysis of quantum algebras, the mathematics literature would support the expectation [12, 13] that the composition of momenta in the two-particle sector should involve a troubling lack of symmetry between pairs of particles. Even in the case of two identical particles it appears necessary to handle to the two momenta in a nonsymmetric way, while the composition of energies is undeformed. On the contrary, the analysis reported in Section 2 shows that consistency with the postulates does not require any such loss of symmetry under exchange of particles. It appears therefore plausible that a mathematical description of the line of analysis advocated in Section 2 may require the introduction of new concepts in the subject of quantum algebras.

The very recent analysis reported in Ref. [5] appears to provide the tools for introducing these new concepts in $\kappa$-Poincaré, and provide solutions for some of the reasons of concern which have traditionally obstructed attempts to apply the $\kappa$-Poincaré formalism in physics. To these observations I devote the remainder of this Section.

### 3.1 Role of the $\kappa$-Poincaré coproduct in particle-production rules

One of the key obstacles for physics applications of the $\kappa$-Poincaré formalism is associated with the $\kappa$-Poincaré coproduct, which readers familiar with $\kappa$-Poincaré will recognize (of course, in leading order) in the "$\dot{+}$" operation here introduced in Subsection 2.3: two momenta, $p$ and $k$, are combined in the coproduct by the rule $p\dot{+}k$. The fact that $p\dot{+}k$ is affected by a severe loss of $p,k$-exchange symmetry has motivated some skepticism toward the applicability of $\kappa$-Poincaré in physics.

For example, in the $\kappa$-Poincaré literature it has been assumed that scattering processes involving two incoming and two outgoing particles should conserve total momentum in the sense that the coproduct sum of the incoming momenta should equal the coproduct sum of the outgoing momenta. This appears troubling since the lack of symmetry of the coproduct would imply, for example, that in the case of two identical particles colliding to produce two other identical particles one should choose which of the incoming momenta enters the coproduct from the left and a similar choice would have to be made for the outgoing particles. This problem is solved by the particle-production rule (cons') here introduced in Subsection 2.3, which involves the coproduct in a way that however does not force us to choose the ordering of the incoming and outgoing momenta: (cons') treats in a fully symmetric way all momenta involved in the process. The price payed for this reassuring result is that (cons') does not admit interpretation as an ordinary rule of energy-momentum conservation: (cons') actually states that, *e.g.*, a process with two incoming and two outgoing particles, rather than having to obey a single fixed conservation rule, can be realized through any one of 24 conservation rules, obtained by permutations of the four momenta involved in the process.

This prediction is rather strikingly new, but it does not pose any conceptual problem for application in physics (no required choice between identical particles) and is actually consistent with all available data, which concern momenta that are much smaller than the inverse of the Planck length (in which case the mentioned 24 particle-production channels all collapse into a single and ordinary conservation rule).

As a first example of application of the rule (cons') let me note here the explicit formulas that according to (cons') would describe the process in which a photon of energy $E$ and a photon of energy $\epsilon$, with $\epsilon \ll E$, collide and produce an electron-positron pair. The analysis of (cons') is rather simple if we assume that the process is at threshold (incoming photons only barely satisfy the energetic requirements for producing the electron-positron pair) and we include only the leading-order corrections, of order $\tilde{L}_p E^2$. In this limit one easily finds that the 24 channels actually all give raise to the same conservation rule (again, I stress that this is the leading-order result). While in conventional physics one would impose the relation $2p = E - \epsilon$ on the common momentum of the produced pair (at threshold the electron and the positron necessarily emerge with identical momenta), according to (cons') one should impose the condition

$2p = E - \epsilon + \tilde{L}_p E^2/4$ (all 24 channels predict this same relation in leading order). Combining this result with the structure of the dispersion relation one finds that there is no leading-order deformation of the threshold condition: the deformation of the dispersion relation is compensated by the deformation of momentum conservation, giving back the ordinary threshold condition $E\epsilon = m_e^2$, with $m_e$ the electron mass.

This cancellation of leading-order corrections to the threshold condition for processes described in a highly boosted frame (a frame which is highly boosted with respect to the center-of-mass frame) may at first appear reassuring. The new Relativity theory (and its underlying $\kappa$-Poincaré mathematics) turn out to be a deformation of conventional Relativity that is even milder than expected (one could expect effects to be small because of the Planck-length suppression, but one might have not guessed that even the leading-order term in the Planck length cancels out). However, it is instead more correct to describe this result of cancellation of leading-order effect as disappointing. In fact, just for processes seen in a "LAB frame" which is highly boosted with respect to the center-of-mass frame there is growing evidence in support of an anomaly in the threshold conditions. This evidence emerges from astrophysical observations which I will discuss in Section 5, but the key point is that in order to explain these observations it would have been useful [14, 17] to encounter a leading-order correction to the threshold conditions.

The next question to ask is of course: Does the cancellation of leading-order corrections to the threshold conditions mean that the new Relativity theory cannot provide an explanation for these puzzling observations? A definite answer to this question still requires additional investigations. The new Relativity theory might still explain the mentioned puzzling observations, but, if it does, the structure of the solution must be more complicated than a simple deformation of the threshold conditions. It seems to me that there are at least three promising avenues to seek a solution of the observational paradoxes within the new Relativity theory: (i) The peak of the cross section for the particle-physics processes [14, 17] relevant for the mentioned observational paradoxes is not exactly at threshold; it is somewhat above threshold. The fact that the leading-order corrections cancel each other out for the very special conditions required by threshold production does not necessarily imply that above threshold one should find a similar cancellation. This should be studied and compared with the observations. (ii) The structure of the new particle-production kinematical requirements of the new Relativity theory may actually combine in non-trivial way when a given process actually involves more than one microscopic process (*e.g.* a small cascade). Again, this may affect the comparison of the new theory with observations. (iii) As I shall emphasize in the next Subsection, it appears that the new Relativity theory requires careful handling of composite particles (particles composed by a few fundamental particles). The mentioned astrophysical observations are believed to involve [14, 17] photons, electrons, protons and pions. In a Relativity theory applicable all the way down to the Planck length it appears even conceivable that photons and electrons (and quarks) might not be fundamental particles, and certainly protons and pions cannot be treated as truly fundamental particles. The observations I report in the next Subsection suggest that, in the new Relativity theory, collisions involving composite particles might behave quite differently from collisions among fundamental particles. This is another ingredient which should be taken into account in seeking an explanation for the mentioned puzzling observations in astrophysics.

### 3.2 Differences between microscopic and macroscopic bodies

In the preceding Subsection I presented a solution for one of the obstacles, concerning particle-production processes, for physics applications of the $\kappa$-Poincaré formalism. The solution was motivated by my attempt of constructing new Relativity theories and the role that $\kappa$-Poincaré might have in some of these new theories. There is another, perhaps even more serious, obstacle for physics applications of the $\kappa$-Poincaré formalism: while deformed dispersion relations of the type $E^2 = c^2p^2 + \tilde{L}_p cEp^2$ are consistent with all available data on fundamental particles (these data concern particles with energy-momentum which is much smaller than $1/\tilde{L}_p$) such a deformation is clearly unacceptable for macroscopic bodies (a macroscopic body easily has energy-momentum that is much greater than $1/\tilde{L}_p$ and our data on macroscopic bodies are clearly inconsistent with the deformed dispersion relation). If a deformed dispersion relation of the type $E^2 = c^2p^2 + \tilde{L}_p cEp^2$ should play a role in physics, clearly its applicability must somehow be confined to systems of one or a few fundamental particles; it should not hold for macroscopic bodies.

In the way in which the $\kappa$-Poincaré formalism has been developed until now there is no room for such a separation between microscopic and macroscopic realms. This is again associated with the role that the coproduct had been assumed to play in previous $\kappa$-Poincaré studies. In fact, it was assumed that the coproduct should characterize the total momentum of a multi-particle system; for example, a system composed of two particles, one with momentum $p$ and the other with momentum $k$, would be characterized by total momentum $p\dot{+}k$ (or $k\dot{+}p$, an alarming choice must be made also in this case) and this is found to lead to total momentum and total energy which transform just like the single-particle energy momentum, *i.e.* following the deformed dispersion relation.

Also in this respect the proposal of (cons') can be used to motivate a solution of the paradox, again inspired by the idea that some of the new Relativity theories of the general type proposed in Ref. [1, 2] might in some way involve the $\kappa$-Poincaré formalism. The point is that the concept of total momentum of a multi-particle system must, as all concepts in physics, be introduced to reflect an operatively-defined property of a physical system. A natural opportunity for attributing a physically meaningful significance to the concept of total momentum is provided by collision processes: we will be able to give operative meaning to the concept of total momentum of two incoming particles if some combination of the energy-momenta of these incoming particles is conserved in the process (the corresponding combination of the outgoing-particles energy-momenta takes the same value). The rules (cons'), which are fully consistent with the new Relativity postulates, do not admit this type of interpretation: they cannot be described as an equality between a sum involving only the incoming momenta and a sum involving only the outgoing momenta. According to (cons') it is in particular not legitimate to take the coproduct sum of the incoming momenta as the total momentum of that two-particle system. This is sufficient to provide the needed opportunity for a separation between microscopic and macroscopic realms: if the total momentum is not identified with the coproduct sum of the momenta it will not necessarily obey the same dispersion relation of the energy-momentum of a single particle. To a system composed of a number $N$ of particles in the new Relativity theory we cannot assign a meaningful "total momentum"; we must keep track of all individual momenta of the composing particles and analyze collision processes from that starting point.

Let me also observe, however, that in some weak sense it is possible to introduce some sort of total momentum. Using again (cons') it appears that we should describe the concept of total momentum only as some sort of average property of a macroscopic body. We will have a good definition of total momentum of a macroscopic body if we identify a characteristic momentum of a macroscopic body which is conserved in collisions between macroscopic bodies. (cons') does not allow to enforce such a condition exactly, but it does allow to introduce such a condition in an appropriate statistical sense. Let us consider the collision of two macroscopic bodies, each composed by an Avogadro number of fundamental particles: such a collision at a fundamental level will actually involve a very large number of collisions between the fundamental particles that compose the macroscopic bodies. Each of these microscopic collisions will actually be characterized by a single one of the 24 channels (when the process is $2 \rightarrow 2$) but the collision between two macroscopic bodies will involve such a large number of these microscopic collisions that it will be characterized by the average of the 24 channels.[9]

At least in leading order in $\tilde{L}_p$, it appears plausible that this authomatic averaging procedure would lead to the introduction of a (non-fundamental) concept of total momentum of a macroscopic body which, just as in conventional physics, is based on the ordinary sum of momenta. This however might only be applicable to collisions between macroscopic bodies whose velocities are not very high, so that a small boost is sufficient to take the system to the center-of-mass frame.

In support for this possibility let me analyze the implications of (cons') for a center-of-mass collision of two identical particles with momenta $p$ and $-p$ respectively, and of course same energy $E$, that produces two other identical particles just above threshold. This microscopic process would be one of the many microscopic processes that occur when two macroscopic bodies collide. Conventional physics would predict that the sum of the momenta of the outgoing particles, $p'_1 + p'_2$, should vanish: $p'_1 + p'_2 = 0$. From (cons') one easily finds that, in leading order[10] in this case the 24 channels that characterize (cons') split up into 8 channels with $p'_1 + p'_2 = 0$, 4 channels with $p'_1 + p'_2 + \tilde{L}_p(E/c)(p'_1 - p'_2)/2 = \tilde{L}_p E p/c$, 4 channels with $p'_1 + p'_2 + \tilde{L}_p(E/c)(p'_1 - p'_2)/2 = -\tilde{L}_p E p/c$, 4 channels with $p'_1 + p'_2 - \tilde{L}_p(E/c)(p'_1 - p'_2)/2 = \tilde{L}_p E p/c$, 4 channels with $p'_1 + p'_2 - \tilde{L}_p(E/c)(p'_1 - p'_2)/2 = -\tilde{L}_p E p/c$. A single process of this type would follow a single one of these 24 options, but a collection of a large number of these processes would be primarily characterized by the average behaviour of the 24 channels, which is simply $< p'_1 + p'_2 >= 0$.

[9] I am of course describing the ingredients of this proposal in a simplified way: in a real situation different types of fundamental particles would compose the macroscopic body and different types of collisions would occur at a fundamental level. However, the main point will still hold; the deformation described by the rules (cons') would be averaged out as a result of the large number of collisions occurring at the fundamental level.

[10] In the previous Subsection I applied (cons') to a microscopic process seen by an observer characterized by a large boost with respect to the center-of-mass frame. In that case the 24 channels agreed in leading order. As shown in the present Subsection, in the description of the same microscopic process by the center-of-mass observer the 24 channels that characterize (cons') do not agree even in leading order. It is easy to check that this difference between center-of-mass observers and highly boosted observers is fully consistent with (and actually reflects the properties of) the deformed boost action.

# 4 Relations with other results of quantum-gravity research

Perhaps the most important implication of the proposal I put forward in Refs. [1, 2] is that it is now clear that there are two alternatives for introducing the Planck length in the fundamental structure of space-time. Before the proposal [1, 2] the only option was provided by the present interpretation of the Planck length as a scale characteristic of the rules of dynamics, just a rescaled value of the gravitational coupling $G$. In that approach the Planck length could enter space-time structure only when accompanied by an associated background (*e.g.* as a scale present in a/the vacuum solution of the equations of dynamics). Indeed, even maintaining the postulates of Special Relativity unmodified (including Fitgerald-Lorentz length contraction), it is of course possible [1, 18, 19, 20] that the Planck length be associated with some sort of background. This would be analogous to the well-known special-relativistic description of the motion of an electron in a background electromagnetic field, which is described by different observers in a way that is consistent with the Relativity Principle, but only when these observers take into account the fact that the background electromagnetic field also takes different values in different inertial frames. The Planck length could play a similar role in space-time structure, *i.e.* it could reflect the properties of a background, but then the presence of such a background would allow to single out a "preferred" class of inertial frames for the description of the short-distance structure of space-time (the "preferred" class of inertial frames would of course be identified using Fitgerald-Lorentz contraction which does not allow the presence of an observer-independent scale in space-time structure).

The results I reported in Refs. [1, 2] show that in addition to this traditional scenario, which introduces the Planck length together with a preferred class of inertial frames, it is also possible to follow another scenario for the introduction of the Planck length. This second option does not predict preferred inertial observers but does require a short-distance deformation of boosts and an associated modification of the Relativity postulates. My intuition that such a scenario should be explored found additional encouragement even after the announcement of Refs. [1, 2], especially through conversations in which I became aware of arguments put forward by other colleagues [21, 22] in support of the hypothesis that we might eventually encounter a deformation of boosts (of course, also those arguments were motivated [21, 22] by quantum-gravity issues, such as minimum length and the quantum mechanics of black holes).

I must also stress that, in light of my results [1, 2], it appears necessary for authors to be more careful in their description of certain popular quantum-gravity concepts, such a "minimum length" and "deformed dispersion relation". In many quantum-gravity approaches [23, 24, 25, 26, 27, 28, 29, 30, 31] one or another formulation of the concept of "minimum length" is discussed. However, these studies do not clarify how the presence of a minimum length could affect boosts. This appears to be a serious omission, since, as emphasized above, there are two options for introducing such concepts: either as a characteristic of quantum geometrodynamics (without any modification of the Special-Relativity postulates) or as a characteristic of the Relativity postulates. Similar issues arise in the analysis of approaches (see, *e.g.*, Refs. [18, 19]) predicting new-physics effects that would be strong for particles of wavelength of the order of the Planck length but would be weak for particles of larger wavelengths, such as the ones associated with deformed dispersion relations. Clearly, assuming ordinary

special-relativistic rules of transformation of energy and momentum, these dispersion relations would allow to select a preferred class of inertial frames, but I have shown that deformed dispersion relations can also be introduced as observer-independent laws, at the price of revising Special Relativity.

Another class of studies which have emerged in more or less direct connection with quantum-gravity research and might be reanalyzed from the perspective advocated in my proposal [1, 2] is the one of deformations of various types of algebras motivated by the desire to implement the existence of concepts such as minimum length, minimum de Broglie wavelength or a maximum accelleration [32, 33, 34, 35, 36]. Again in these studies until now much emphasis has been placed on the algebraic tools, but the readers were left without any explicit remarks concerning the faith of the Special-Relativity postulates. It would be interesting to reanalyse the relevant proposals within the new relativistic conceptual framework here proposed, particularly working toward the identification of transformation rules such that the equations describing minimum length and/or minimum de Broglie wavelength and/or maximum accelleration acquire the status of being observer-independent (valid in every inertial frame). What are then the new relativistic transformation rules between observers? Are they physically acceptable? (For example, do the new Lorentz transformations form group, or just a quasigroup?)

# 5 Closing remarks

This closing Section is devoted to a summary of the main results discussed in the previous Sections and to the discussion of some developments of this new research line which, in my opinion, deserve urgent attention.

## 5.1 Relativity can be doubly special

From the viewpoint advocated here and in Refs.[1, 2] the Relativity Principle is somewhat hostile to the introduction of observer-idependent physical scales. In that respect, Einstein's Relativity postulates well deserve to be qualified as "special", since they provide an example in which the Relativity Principle coexists with an observer-independent (velocity) scale. My studies have shown that one can also consistently construct a "Doubly Special Relativity", in which the Relativity Principle coexists with observer-independent scales of both length (or momentum) and velocity. If this option, now shown to be viable, turns out to be chosen by Nature we would have a sort of "Quantum Special Relativity", in the sense encoded in the role played by the Planck length.

## 5.2 Toward a new approach to Quantum Gravity

While the motivation for my studies comes from the desire to eventually unify General Relativity and Quantum Mechanics, the approach is at present still only able to handle flat space-time. In a sense I have constructed a "Quantum Special Relativity" but the natural ultimate goal of this research programme should be a "Quantum General Relativity".

In working toward this ultimate objective a useful intermediate step could be the one of applying the new postulates in contexts with a curved, but still fixed (non-dynamical), space-time, such as De Sitter or Schwarzschild.

Another interesting possibility is the one of describing space-time curvature as a requirement of non-commuting momenta[11]. If this viewpoint turned out to be correct, one could perhaps reach the formulation of a "Quantum General Relativity" by an appropriate extension of the $\kappa$-Minkowski space-time (2) to some sort of "$\kappa$" phase space (which however here is intended as the space $x_i, t, p_i, E$ rather than just $x_i, p_i$).

Even before these preliminary steps are done, there are certain conceptual issues that must be analyzed. One key point is that at present the Planck length $L_p$ is seen as a quantity which is derived from three fundamental constants $c$, $G$ and $\hbar$. The fundamental constants $c$, $G$ and $\hbar$ already have their own operative definitions ($c$ can be measured as the speed of long-wavelength photons, $G$ can be measured, *e.g.*, by simple studies of the solar system, and $\hbar$ can be measured by combined analysis of data on quantum effects, *e.g.*, the black-body spectrum and the photoelectric effect). If $L_p$ is introduced in the Relativity postulates then $L_p$ is authomatically promoted from the status of derived constant to the status of fundamental constant (since it would then have its own operative definition, as in (law2)). The relation between $L_p$, $c$, $G$ and $\hbar$ would accordingly acquire the very rare status of a relation between fundamental concepts, all with their own operative definition. This is very rare in physics, but it cannot be excluded since we have at least one example in which something like this happens: the inertial mass and the gravitational mass of a particle are two concepts with independent operative definitions, but there is a relation between them (they are equal to each other because of the Equivalence Principle). Therefore one way to conceptualize my proposal [1, 2] would require a non-trivial step, somewhat analogous to the introduction of the Equivalence Principle. There is of course another (perhaps even more radical) conceptual alternative: somehow this formalism that provides an intrinsic operative definition for the Planck length might eventually lead to the understanding of one of the two scales not explicitly present in the new Relativity postulates, either $G$ or $\hbar$, as a derived concept. This is a fascinating possibility, which however might require surprising discoveries in the development of the formalism.

## 5.3 Phenomenology

It is important to notice that the possibility of new Relativity postulates involving the Planck length is not merely of academic interest. On the contrary, as shown by the illustrative example of new postulates on which I focused, there can be significant phenomenological implications. These implications have been discussed in some detail in Ref. [1].

The deformation of the dispersion relation introduced in the postulate (law2) can be tested [18, 37, 38, 39] with forthcoming experiments, even if the deformation scale $\tilde{L}_p$ is indeed of the order of the tiny Planck length $\tilde{L}_p \sim L_p$.

The deformed rules for particle production (here discussed in Subsection 2.3) could be most effectively tested in experiments sensitive to the structure of the threshold

[11] An important role in my interest in this possibility was played by exciting discussions with Shahn Majid at the time of our collaboration for the study [15].

requirements for particle production. Interestingly, some of these experiments, observations of ultra-high-energy cosmic rays [40] and of Markarian501 photons [41], have recently obtained data that appear to be in conflict with conventional theories and appear to require [42, 43, 44, 45, 14, 17] a deformation of the kinematic conservation rules applied to collision processes. These observations clearly provide some encouragement for the idea of new Relativity postulates. As I emphasized in Subsection 3.1, the illustrative example of new postulates on which I focused appears to provide an avenue for explaining these paradoxical observations, but more work is needed in order to substantiate this hypothesis. At a preliminary level of analysis I found (in Ref. [1] and here in Subsection 3.1) a result which does not provide clear encouragement for this hypothesis: I found two corrections to the threshold conditions, and the corrections do have exactly the right magnitude to explain the observational paradoxes, but the two corrections cancel each other out (to the relevant order). In Ref. [1] and here in Subsection 3.1 I identified certain alternative mechanisms for explaining the paradoxes within the new Relativity theory. A detailed analysis of these alternative mechanisms is postponed to future studies. This is perhaps the most exciting and urgent issue for the development of the new Relativity theory.

A third class of phenomenological studies that could be significantly affected by the new Relativity postulates is the one pertaining to cosmology and the early stages of evolution of the Universe. The interested reader can find brief remarks on this point in Ref. [2] and a more detailed (and preliminarily quantitative) study in Ref. [4].

## 5.4 Other forms of new Relativity postulates

My proposal [1, 2] of exploring the possibility of new Relativity postulates, involving the Planck length, could of course be followed investigating a large variety of classes of new postulates; however, until now all results have been obtained within the one illustrative example on which I focused also in this paper.

It would be interesting to explore a few alternative possibilities. If nothing else, alternative choices of the postulates could clarify whether or not the satisfactory outcome of the consistency tests of the illustrative example considered in these first studies is significant. If it turned out that other choices of this new type of postulates lead to some inconsistencies, the illustrative example which has proven to have such nice properties could be seen as a strong candidate (while at present it must be only considered as a first example of consistent new postulates). In this respect a key point might emerge from the analysis of combinations of boosts. Whereas in the illustrative example pursued until now the new Lorentz transformations form group in the ordinary sense, it appears plausible that other choices of the new postulates would only lead to quasigroup structure [16], a rather undesireable feature.

Another potentially interesting possibility is the one of attempting to introduce even a third observer-independent scale in the postulates. Since my results showed that a logically consistent framework can emerge from Relativity postulates with a second observer-independent scale, it is now natural to wonder whether a third observer-independent scale could also be consistently introduced. Motivated by the studies I reported in Refs. [1, 2], Kowalski-Glikman has briefly presented in Ref. [3] some "aestethic arguments" (not guided by experimental input or by conceptual urgency, but by an intuition for the conceptual elegance of the fundamental laws of physics) in favour of Relativity with three observer-independent scales, but did not formulate

any attempt to provide an operative definition of the third scale (the entire analysis reported in Ref. [3] relies on the type of deformation of the dispersion relation which I had introduced with (law2)). Consistently with the explorative spirit of my proposal [1, 2], I neither favour nor disfavour *a priori* any particular number of observer-independent scales. We know experimentally that there is at last one observer-independent scale, $c$, in the Relativity postulates. My studies have shown that a second observer-independent scale can be consistently introduced, but we now must wait for the virdict of experimental tests. Examples of Relativity postulates with three observer-independent scales should be studied, and, if any class of such postulates turned out to be logically consistent, corresponding experimental tests are certainly well motivated. In this respect I should emphasize that, as discussed in Ref. [1], for each observer-independent scale the postulates should also provide an operative definition of that scale (this key point was omitted in Ref. [3]). Also important are some considerations related with the remarks I made above in Subsection 5.2. In promoting the Planck length from the status of derived scale to the status of fundamental scale (with its own independent operative definition) we are faced with significant (but exciting) conceptual challenges, and of course attempts of using my proposal [1, 2] in the direction of introducing even a third observer-independent scale are confronted by conceptual challenges which are even more serious (*e.g.*, if the three scales in the postulates are related with $c$ and two combinations of the other scales $G$ and $\hbar$, we could even contemplate the possibility of interpreting both $G$ and $\hbar$ as derived scales!).

## 5.5 Understanding $\kappa$-Poincaré

The illustrative example of new Relativity postulates on which I focused ended up making strong contact with pre-existing mathematics of $\kappa$-Poincaré algebras, first developed through pioneering studies of Lukierski, Ruegg and collaborators [10, 11] (although the relevant example of $\kappa$-Poincaré algebra was discovered more recently [12, 13]). While waiting for experimental tests, one can only hope that somehow this connection with pre-existing mathematics might be a good auspice for the fortunes of this type of new Relativity postulates, just like the introduction, nearly a century ago, of the Special-Relativity postulates, which turned out to lead to pre-existing Lorentz mathematics, was blessed by a long string of experimental successes.

While pre-existing Lorents mathematics really provided all the tools needed for analyses based on Special Relativity, my proposal was confronted [1, 2] with some missing pieces in the development of $\kappa$-Poincaré, particularly the lack of understanding of the role of the coproduct in the laws for particle production and the lack of the needed mechanism for confining the applicability of the $\kappa$-Poincaré dispersion relation to the microscopic realm. The new Relativity postulates led me to propose some solutions for these outstanding problems of $\kappa$-Poincaré. In Subsection 3.1, using results obtained in Refs. [1, 5], I argued that the are some consistent ways to introduce the $\kappa$-Poincaré coproduct in particle-production rules, without any of the feared problems associated with a lack of symmetry under exchange of the momenta of identical particles. In Subsection 3.2, using again results obtained in Ref. [5], I argued that the concept of "total momentum of a multi-particle body" is a highly non-trivial concept in $\kappa$-Poincaré, and that this allows to find ways to confine the applicability of the $\kappa$-Poincaré dispersion relation to the microscopic realm.

These results, besides playing a key role in my new type of Relativity theories, appear to have even wider significance, possibly of use in all contexts in which $\kappa$-Poincaré is being considered as a useful mathematical structure. The two problems solved in Subsections 3.1 and 3.2 were the most alarming residual problems in the conceptual analysis of $\kappa$-Poincaré. Their solution should perhaps reenergize research aimed at addressing the residual technical challenges of $\kappa$-Poincaré, particularly the identification of a natural measure for integration over energy-momentum space [15, 5].


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